



Contents

How to Use This Book	iii
----------------------	-----

Acknowledgements	vii
------------------	-----

1	Counting Is Arithmetic	1
1.1	Introduction	1
1.2	Counting Lists of Numbers	2
1.3	Counting with Addition and Subtraction	6
1.4	Counting Multiple Events	13
1.5	Permutations	19
1.6	Summary	21

2	Basic Counting Techniques	27
2.1	Introduction	27
2.2	Casework	27
2.3	Complementary Counting	35
2.4	Constructive Counting	38
2.5	Counting with Restrictions	41
2.6	Summary	45

3	Correcting for Overcounting	49
3.1	Introduction	49
3.2	Permutations with Repeated Elements	49
3.3	Counting Pairs of Items	53

3.4	Counting with Symmetries	57
3.5	Summary	62

4 Committees and Combinations 65

4.1	Introduction	65
4.2	Committee Forming	65
4.3	How to Compute Combinations	69
4.4	Our First Combinatorial Identity	72
4.5	Summary	76

5 More With Combinations 81

5.1	Introduction	81
5.2	Paths on a Grid	81
5.3	More Committee-type Problems	84
5.4	Distinguishability	88
5.5	Summary	92

6 Some Harder Counting Problems 97

6.1	Introduction	97
6.2	Problems	97
6.3	Summary	108

7 Introduction to Probability 111

7.1	Introduction	111
7.2	Basic Probability	112
7.3	Equally Likely Outcomes	115
7.4	Counting Techniques in Probability Problems	118
7.5	Summary	120

8 Basic Probability Techniques 123

8.1	Introduction	123
8.2	Probability and Addition	123
8.3	Complementary Probabilities	128

8.4	Probability and Multiplication	130
8.5	Probability with Dependent Events	136
8.6★	Shooting Stars – a hard problem	138
8.7	Summary	141

9 Think About It! 145

9.1	Introduction	145
9.2	Problems	145
9.3	Summary	150

10 Geometric Probability 153

10.1	Introduction	153
10.2	Probability Using Lengths	154
10.3	Probability Using Areas	156
10.4	Summary	162

11 Expected Value 165

11.1	Introduction	165
11.2	Definition of Expected Value	166
11.3	Expected Value Problems	166
11.4★	A Funky Game	169
11.5	Summary	171

12 Pascal's Triangle 175

12.1	Introduction	175
12.2	Constructing Pascal's Triangle	176
12.3	Those Numbers Look Familiar!	177
12.4	An Interesting Combinatorial Identity	180
12.5	Another Interesting Combinatorial Identity	184
12.6	Summary	187

13 The Hockey Stick Identity 191

13.1	Introduction	191
------	------------------------	-----

13.2	The Problem	192
13.3	A Step-by-Step Solution	193
13.4	A Clever Solution	198
13.5	The Identity	199
13.6	Summary	205

14	The Binomial Theorem	209
14.1	Introduction	209
14.2	A Little Algebra	209
14.3	The Theorem	211
14.4	Applications of the Binomial Theorem	213
14.5	Using the Binomial Theorem in Identities	214
14.6	Summary	216

15	More Challenging Problems	219
15.1	Introduction	219
15.2	Problems	219

Hints to Selected Problems	231
-----------------------------------	------------

Index	241
--------------	------------
