

Contents

0 Prove It!	1
1 Logarithms	3
2 Not Just For Right Triangles	10
2.1 Trigonometric Functions	10
2.2 Graphing Trigonometric Functions	12
2.3 Going Backwards	14
2.4 Tying It All Together	15
2.5 Solving Problems Using Trigonometric Identities	20
3 More Triangles!	24
3.1 Triangle Laws	24
3.2 Areas, Areas, Areas	27
3.3 More Important Lines	29
4 Cyclic Quadrilaterals	33
4.1 Properties of Cyclic Quadrilaterals	33
4.2 Finding Cyclic Quadrilaterals	33
4.3 Ptolemy's Theorem	35
5 Conics and Polar Coordinates	38
5.1 Parabolas	38
5.2 Ellipses	40
5.3 Hyperbolas	44
5.4 Polar Coordinates Revisited	47
5.5 That Pesky xy Term	49
6 Polynomials	52
6.1 What is a Polynomial?	52
6.2 Multiplying and Dividing Polynomials	52
6.3 Finding Roots of Polynomials	56
6.4 Coefficients and Roots	60

6.5	Transforming Polynomials	62
6.6	Newton's Sums	65
7	Functions	69
7.1	The Inverse of a Function	69
7.2	Functional Identities	70
7.3	Solving Functional Identities	71
7.3.1	Isolation	71
7.3.2	Substituting in Values	72
7.3.3	Using Cyclic Functions	72
7.3.4	Arbitrary Functions	73
8	Taking it to the Limit	76
8.1	What is a Limit?	76
8.2	Tricky	77
8.3	Working with Limits	78
8.4	Continuity	80
8.5	Asymptotes	81
8.6	Trig Limits	83
8.7	e	84
9	Complex Numbers	88
9.1	Drawing the Complex Numbers	88
9.2	The Complex Absolute Value	89
9.3	Complex Multiplication and Coordinates	91
9.4	Complex Powers and Geometry	91
9.5	DeMoivre's Theorem	92
9.6	Exponential Form	94
9.7	Two for One	96
9.8	The Roots of Unity	96
10	Vectors and Matrices	100
10.1	What is a Vector?	100
10.2	The Dot Product	101
10.3	Coordinate Representation of Vectors	102
10.4	What is a Matrix?	103
10.5	Matrix Multiplication	104
10.6	Matrices in Higher Dimensions	107
10.7	Better Matrix Notation	108
11	Cross Products and Determinants	112

11.1	The Cross Product	112
11.2	The Cross Product in Coordinates	113
11.3	The Determinant	113
11.4	Determinants in Higher Dimensions	115
11.5	Minors	116
11.6	Row and Column Operations	118
11.7	The Inverse of a Matrix	120
12	Analytic Geometry	124
12.1	Lines, Angles, and Distances	124
12.2	Parameters	125
12.3	Vectors	127
12.4	Points, Lines, and Planes	130
12.5	Curved Surfaces	134
12.6	Using Analytic Geometry	135
12.7	Vectors and Geometry Problems	136
13	Equations and Expressions	143
13.1	Linear Equations	143
13.2	Convenient Systems	148
13.3	Symmetric Expressions and Advanced Factorizations	150
13.4	More Polynomials	152
13.5	Squares and Cubes	154
13.6	Using Graphing	155
14	Inequalities	159
14.1	Trivial Inequality Revisited	159
14.2	Arithmetic Mean-Geometric Mean Inequality	160
14.3	Cauchy's Inequality	162
14.4	Maximization and Minimization	164
14.5	Geometry and Inequalities	165
14.6	Wrap-Up and Parting Hints	166
15	Combinatorics	170
15.1	Identities	170
15.2	Pascal's Identity	170
15.3	More Identities	172
15.4	Block Walking	173
15.5	The Binomial Theorem	175
16	Sequences and Series	180

16.1	Fractions in Other Bases	180
16.2	Some Special Series	181
16.3	The Fibonacci Numbers	182
16.4	Dealing with Recurrences	184
16.5	Dealing with Sums	186
16.6	The Binomial Theorem Revisited	188
16.7	Harmonic Sequences	191
17	Counting in the Twilight Zone	196
17.1	One to One	196
17.2	Clever Correspondences	196
17.3	Easy as	198
17.4	Generating Functions	200
17.5	Partitions	203
17.6	Counting on Graphs	204
17.7	Counting Infinite Sets	205
18	Again and Again	211
18.1	Repeats	211
18.2	Off to Infinity	211
18.3	Rational Continued Fractions	212
18.4	Real Continued Fractions	213
19	Probability	216
19.1	Review, Definitions, and Notation	216
19.2	Going a Step Further	217
19.3	Geometry and Probability	217
19.4	Conditional Probability	219
20	Find It and Make It	224
20.1	Locus	224
20.2	Construction	227
21	Collinearity and Concurrency	233
21.1	Three Points and a Line	233
21.2	Three Lines and a Point	236
22	Geometry Tidbits	241
22.1	Projections	241
22.2	Inversion	243
22.3	Homothety	245
22.4	Geometric Continuity	246

22.5 Given a Finite Number of	247
23 Number Theory	252
23.1 Divisibility	252
23.2 Division in Congruences	253
23.3 Solving Linear Congruences	254
23.4 Solving Quadratic Congruences	256
23.5 The Sum of the Divisors	257
23.6 Fermat's Theorem	258
23.7 The ϕ Function	260
23.8 Wilson's Theorem	262
24 Diophantine Equations	266
24.1 $ax + by = c$	266
24.1.1 $c = 0$	266
24.1.2 $c \neq 0$	267
24.2 $x^2 + y^2 = z^2$	269
24.3 $x^4 + y^4 = z^2$	270
24.4 The Pell Equation	271
24.5 General Methods	272
25 Graph Theory	275
25.1 Points and Lines	275
25.2 Planar Graphs	276
25.3 Example: The Platonic Solids	277
25.4 Walking Around on Graphs	278
25.5 Euler Trails	279
25.6 Colorings	280
26 Parting Shots	284